Chapter 5. Determination of the Vp/Vs Ratio

If the P and S phases of an earthquake are read at two or more stations and if the ratio of the P velocity to the S velocity (Vp/Vs) is assumed to be a constant, then the Vp/Vs ratio and origin time can be computed. If P- and S-arrival pairs are available from more than two stations, then in general a plot of S-arrival time versus P-arrival time will not define a single straight line, so some "best fitting" line must be found in order to estimate the slope (Vp/Vs) and the origin time. Since both the P- and the S-arrival times are subject to random errors, it is not appropriate to compute the least-square regression of P on S (which assumes S has no errors) or the regression of S on P (which assumes P has no errors). Instead, errors in both P and S should be taken into account by minimizing (Madansky, 1959)

$$T = \sum_1^n W_i (S_i - A - BP_i)^2 \text{, where}$$

$$B = \frac{Vp}{Vs}\text{, } A \text{ is the } S \text{ axis intercept, and}$$

for the i[th] of n stations:

$$S_i \text{ is the } S \text{ arrival time}$$

$$P_i \text{ is the } P \text{ arrival time}$$

$$W_i = \frac{1}{E_{S_i}^2 + B\,E_{P_i}^2}$$

$$E_{S_i} = C \text{ times the standard error of the } S \text{ arrival time}$$

$$E_{P_i} = C \text{ times the standard error of the } P \text{ arrival time}$$

[The standard errors are computed from the assigned weight codes for P and S. C is an arbitrary constant.]

Although there are closed solutions to this problem if the standard errors (or variances) of S and P are the same for all i (Madansky, 1959), an iterative technique was developed for use with seismic data for which the variance is estimated for each reading. In computing the sum to be minimized for a given value of B, the data are first centered by subtracting the weighted mean of the S arrivals from each S-arrival time and the weighted mean of the P arrivals from each P-arrival time:

$$SC_i = S_i - \frac{\sum W_i S_i}{\sum W_i} \qquad PC_i = P_i - \frac{\sum W_i P_i}{\sum W_i}$$

then the sum (T) to be minimized is computed from:

$$T = \sum W_i \, (SC_i - B \; PC_i)^2$$

To find the value of B that minimizes T, T is initially computed for five values of B defined by:

$$B_k = BL_1 - DB_1(K - 3), \quad k = 1 \text{ to } 5$$

where $DB_1 = 0.6$ and BL_1 is the average of the weighted least squares regression slope of S on P and the inverse of the weighted regression slope of P on S. T is then compared for the five values of B defined by

$$B_k = BL_2 - DB_2(k - 3), \quad k = 1 \text{ to } 5$$

where $DB_2 = 0.4 \; DB_1$ and BL_2 is the value of B_k which gave the minimum sum T in the previous step. This process is repeated 6 more times, so that B is resolved to the nearest 0.001 units.

The printed output includes the Vp/Vs ratio and the standard error of the Vp/Vs ratio computed from S regressed on P and also from P regressed on S. The standard error of the slope computed using both P and S weights is estimated from the square root of the sum of squares of the standard deviations computed for S regressed on P and for P regressed on S.